

Anomalous Higgs Couplings as a Window to New Physics

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ABSTRACT: Recent data on the production and decay of the Higgs boson suggest that, for a good fit to the various decay channels, deviations from the Standard Model (SM) values of the Higgs coupling with $t\bar{t}$, W^+W^- , and ZZ are indicated. While this itself is hardly a proof of any beyond-SM physics right now, and while the data can very well change in near future, this opens up an interesting avenue to explore regarding unitarity of gauge boson scattering and the stability of the electroweak vacuum in the presence of anomalous couplings. We show that for some benchmark points, touted as the best-fit points in the literature, unitarity in gauge boson scattering breaks down between 1 and 10 TeV. We also show that if there are no new light degrees of freedom, the Higgs quartic coupling becomes negative at around the same point, making the electroweak vacuum unstable. Thus, some new ultraviolet completing new physics is demanded at that scale to cancel both these anomalous behaviours.

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1 Introduction

The recent discovery of a resonance by both the ATLAS [1] and the CMS [2] collaborations at the Large Hadron Collider (LHC) has led to intense activity. This has been accentuated by the fact that both the groups report excesses—over the Standard Model (SM) backgrounds—in multiple channels and concentrated at nearly the same (125–126 GeV) reconstructed mass. Supported by evidence from the Tevatron [3], this has naturally led to euphoria in the community. However, even though the resonance is obviously a boson, its identification with the long-awaited Higgs particle of the standard electroweak theory is not yet certain.

While the observed diphoton decay mode excludes the possibility of the said resonance being a spin-1 particle, other non-trivial assignments are, as yet, possible. This is notwithstanding some preliminary reports that favour a $J^P = 0^+$ assignment. Furthermore, even if the said assignment is proven, that still does not uniquely identify the observed particle to be *the* SM Higgs boson. For one, the observed excesses do not precisely match the rates expected within the SM. In particular, the diphoton (the channel permitting the cleanest measurement of the mass) rate is significantly above the expectations, while the $\tau^+\tau^-$ channel seems to be somewhat suppressed. While these are yet early days to claim a discrepancy, the observed patterns have led to intense speculations about the nature of

this particle and the ramifications of this discovery for a host of scenarios of physics beyond the Standard Model (BSM) [4–6]. At this juncture, it must be recognised that the purported discrepancies could just be a manifestation of the inherent uncertainties in QCD calculations, both in the perturbative and the non-perturbative regimes [7]. On the other hand, ratios of signal rates (such as that between the diphoton and the four-lepton final states) are relatively free of such uncertainties and constitute a more robust signature of a deviation from the SM expectations [8]. A framework for this has been discussed in Ref. [9], where the scale factors g_X , parametrizing the deviation of Higgs decay width or production cross section, were defined; the production cross section σ_{XX} for $XX \rightarrow h$ or the decay width Γ_{XX} for $h \rightarrow XX$ has an extra multiplicative scale factor g_X^2 when compared with the SM predictions.

In view of the state of affairs (compounded by the fact that no other distinct departure from the SM has been observed at the LHC), the authors of Ref. [6] effected an interesting phenomenological study. Considering all of the Higgs couplings, whether tree-level or loop-induced, to be unrelated and free parameters, as well as allowing for an invisible decay mode for the Higgs, they used the observations (both low-energy, such as precision electroweak observables, as well as the recent data) to obtain a best fit to the same. In as much as no underlying physics assumptions (other than Lorentz invariance) were made for this sector (*i.e.*, no patterns were imposed on the anomalous couplings of the Higgs, whether to the gauge bosons or to the fermions), this constitutes, perhaps, the most general investigation to the possible nature of physics just beyond the SM scale.

One might parametrize the Higgs effective coupling to $t\bar{t}$ and gauge bosons to be

$$\mathcal{L}^{eff} = e^{i\delta} g_t \frac{\sqrt{2} m_t}{v} h \bar{t} t + g_W \frac{2m_W^2}{v} h W_\mu^+ W^{\mu-} + g_Z \frac{m_Z^2}{v} h Z_\mu Z^\mu, \quad (1.1)$$

where v is the vacuum expectation value (VEV) of the Higgs. Within the SM, g_t, g_W and g_Z all equal unity. Allowing these couplings (as also others, which are not germane to the discussions here) to vary independently, Ref. [6] finds that the “best fits”, according to data available in early July, are given by

$$\begin{aligned} (F1) : & \quad g_t = -0.6 & g_W = 1.2 & g_Z = 1.6 \\ (F2) : & \quad g_t = -1.3 & g_W = 1.07 & g_Z = 1.07 \\ (F3) : & \quad g_t = -1.05 e^{0.55 i} & g_W = 1.06 & g_Z = 1.06 . \end{aligned} \quad (1.2)$$

The fits $F1$ & $F2$ were performed holding g_t to be real. Similarly, $F2$ & $F3$ demanded custodial symmetry. The constraint on custodial symmetry breaking, given by the oblique parameter T , is so strong that it is natural to impose $g_W = g_Z$. However, the best fit in $F1$ can be intuitively understood from the fact that all the experiments show the hint of a shortfall in $h \rightarrow W^+ W^-$ channel, while, on the average, $h \rightarrow ZZ$ is closer to the SM prediction¹. Similarly, the excess in $h \rightarrow \gamma\gamma$ can be explained if one can somehow arrange for

¹It is tempting to ascribe the reconciliation of the said shortfall in the WW channel with $g_W > 1$ to the fact that, even for the SM Higgs, gauge boson fusion contributes non-negligibly to h -production and, for the current level of statistics, remains virtually indistinguishable from the dominant gluon-fusion channel. More importantly, though, a best fit to data is weighed by the uncertainties in each channel, and, consequently, the $h \rightarrow W^+ W^-$ mode is accorded relatively lower weight.

a constructive interference between the top-mediated and the W -mediated triangle diagrams (in place of a destructive interference as in the SM), and this can be achieved if the effective $ht\bar{t}$ coupling flips its sign. The effects of such anomalous top Yukawa coupling already received attention much before the discovery of the Higgs boson, *e.g.*, in the context of baryogenesis [10] or unitarity violation in gauge boson scattering [11]. The latter will be particularly relevant for our subsequent discussion.

As the exact nature of the “best fit” would change once more data is included in the fit, we do not consider the cases of Eq.(1.2) to be sacrosanct, but treat them only as indicative of such fits. It is worth noting that once custodial symmetry is imposed, the deviations from the SM, viz. $\delta g_{W,Z}$ ($\equiv g_{W/Z} - 1$) are much smaller than δg_t (this also holds, albeit weakly, for $F1$). Although still larger than what naive dimensional analysis would suggest (for a new physics scale $\gtrsim 500$ GeV), such $\delta g_{W,Z}$ could, presumably, be the result of quantum corrections (possibly, though, in a theory that is either strongly coupled or has a non-trivial ultraviolet completion). The large change of δg_t is, however, a more complicated story and constitutes the bulk of this paper.

One notes that such a change is also indicated in ATLAS and CMS analyses [12, 13] based on their data and the formalism developed in Ref. [9]. Taking $g_t = g_b = g_\tau$ and $g_W = g_Z$ (so that the custodial symmetry is respected), the ATLAS Collaboration found, within 68% confidence limit,

$$g_t \in [-1.0, -0.7] \cup [0.7, 1.3], \quad g_W \in [0.9, 1.0] \cup [1.1, 1.3]. \quad (1.3)$$

The CMS Collaboration, on the other hand, found the best fit at $(g_t, g_V) \approx (-0.7, 0.9)$. However, each of the two collaborations analysed only their own data set, and also did not consider the possibility that the Higgs could decay into any new particles. Thus, the fit in [6] encompasses a wider amount of data. With this caveat, it is easy to appreciate the relatively minor differences in the fits.

While the process of pinning down the various couplings of the Higgs continues as data pour in, it is also necessary to subject our observations to theoretical consistency checks. For example, one important role of the Higgs boson is to ensure partial wave unitarity in various $2 \rightarrow 2$ scattering processes. If the couplings of the Higgs turn out to have non-standard values, then the fine balance required for unitarity is destroyed, and one has to set a cut-off scale for the theory. In this work, we derive values of this cut-off scale for various levels of departure of the Higgs-fermion-antifermion interactions from their standard values. Side by side, we also examine the implications of such modified interaction strengths on the issue of vacuum stability (essentially arising from the radiatively corrected quartic coupling potentially turning negative). And, based on the above considerations, we make some remarks on how the existence of additional particles can restore balance to the whole scenario, if indeed the recently observed scalar has anomalous coupling strengths.

The rest of the paper is arranged as follows. In Section 2, we discuss some theoretical issues pertaining to the choice of these best-fit points; in particular, we would like to spend some time on the point $F3$, which includes a nontrivial phase in the top Yukawa coupling. In Section 3, we discuss the unitarity of $WW \rightarrow t\bar{t}$ and $ZZ \rightarrow t\bar{t}$ scattering with such benchmark points. The evolution of the scalar quartic coupling is discussed in Section 4.

While we do not go into details about models that can produce such effective couplings, some relevant remarks are made in Section 5. We summarize and conclude in the Section 6. Some calculational details as well as a compendium of necessary formulae are put in the appendices.

2 Some Theoretical Issues

Let us first make a few comments on the point *F3*, where a complex top Yukawa coupling is indicated. This immediately raises very pertinent and interesting questions as to the possible sources of such an anomalous coupling. While mixing effects (whether in the Higgs sector or in the fermion sector) can and do cause significant deviations in the coupling, the magnitude of the deviation is never so large unless the new states are both very light and have complicated quantum number assignments. Similarly, such a large anomalous coupling is not expected from loop-corrections (owing to some as-yet-unobserved states) unless the said sector couples very strongly to the observed one². In particular, the existence of a non-zero δ in Eq. (1.1) ostensibly renders the Hamiltonian to be non-Hermitian. As is well-known, such an absorptive part can arise from loop-corrections within a Hermitian theory if there exists an intermediate state that can be on-shell. However, the existence of such a state begs many questions. For one, such particles would necessarily be light and should have manifested themselves not only in Higgs decays, but also in other collider processes. This is particularly so, for, by definition, such a state would be part of an $SU(2)_L$ doublet, which, in turn, would immediately call for similar contributions to the absorptive parts of other effective vertices (with or without the Higgs). Not only this, such a light state should have been produced directly too. In particular, the $SU(2)_L$ antecedents would have required that they be produced at a clean environment such as LEP-II (as also the Tevatron). No signs of either such production, or the inducing of absorptive parts in other couplings have yet been observed.

It is interesting to note that a nonzero phase had been introduced earlier in the top Yukawa coupling, albeit in a different context [10]. Wishing to incorporate CP violation in this interaction (motivated by a desire to address baryogenesis), the authors of Ref. [10] augmented the SM Lagrangian by an effective operator of the form

$$\delta\mathcal{L} = c_\phi e^{i\xi} \bar{Q}_L t_R \Phi + h.c \quad (2.1)$$

where c_ϕ denotes a (real) effective coupling owing its origin to higher-dimension terms. In the unitary gauge, this yields

$$c_\phi \bar{t} [\cos \xi + i \sin \xi \gamma_5] t h \quad (2.2)$$

over and above the SM term. Clearly, a non-zero ξ leads to CP violation. This coupling, though, is markedly different from the ansatz of Ref. [6], as it emanates from an Hermitian effective Lagrangian unlike in the other case. Furthermore, the pseudoscalar term (which,

²This observation applies equally to fit *F1* as well as to g_t of *F2*.

essentially, is the only one to see a non-zero value of the phase ξ) in the coupling above contributes only incoherently to $h \rightarrow \gamma\gamma$ and is, thus, of little consequence.

It should also be realized that, for $m_h \sim 125 \text{ GeV}$, the top quark lines at this vertex cannot be on the positive energy mass-shell. Thus, the application of Cutkosky rules is not straightforward; nor is the identification of δ with the discontinuity across a cut arising from a physical region singularity. In other words, the existence of a non-zero δ in Eq.(1.1) is a subtle issue in quantum field theory and we shall, henceforth, consider the top Yukawa coupling to be real, albeit admitting the possibility of an anomalous component to it. Thus, our benchmark point *F3* will be parametrized by $(g_t, \delta) = (-1.05, 0)$. Note that this does not invalidate Ref. [6], for when they hold $\delta = 0$, they still find that the best fit requires $g_t \neq 1$ with the deviation from the SM being substantial³.

An anomalous top Yukawa coupling (even if real) brings in its own complications. Within the SM, all couplings are dictated by gauge invariance⁴. While deviations are indeed possible once one enlarges the ambit of the theory, gauge invariance would require that these either be associated with higher-dimensional effective operators, or be the consequence of mixings between states (were new states to be admitted). Each of these eventualities would imply correlated deviations in other couplings, and, on occasions, the introduction of new ones. Any uncorrelated deviation, such as that of Eq.(1.1) can only be the result of an additional term in the Lagrangian of the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{anom}} \quad \mathcal{L}_{\text{anom}} = (g_t - 1) \frac{\sqrt{2} m_t}{v} h \bar{t} t \quad (2.3)$$

where, for simplicity, we have chosen $\delta = 0$. Such a term, of course, explicitly breaks $SU(2)_L \otimes U(1)_Y$. While its inclusion may seem to militate against the gauge dogma, note that Eq.(2.3) could just represent the relevant part of the BSM physics, with other terms being hidden for unknown reasons. Thus, the breaking of gauge invariance might be an artefact of restricting ourselves to be close to the augmented SM, which acts only as a low-energy effective theory, while gauge invariance is again restored when we go to the full theory at a high energy. In the effective theory, due to the apparent loss of gauge invariance, the mass and the Yukawa coupling of the fermions, in particular the top quark, get decoupled, and this apparent loss has other profound implications. As is well known, unitarity in gauge boson scattering (in particular, the longitudinal modes) is inextricably linked to gauge invariance. While any loss of unitarity due to $\mathcal{L}_{\text{anom}}$ could, in principle, be restored on inclusion of other terms in \mathcal{L}_{eff} , the scale at which such a loss is seen (if one considers $\mathcal{L}_{\text{anom}}$ alone) would point to the scale of the new theory that underlies such a deviation. Similarly, the existence of $\mathcal{L}_{\text{anom}}$ would have non-trivial consequences for the renormalization group evolution of the couplings in the theory as well for considerations such as the stability of the vacuum.

³Indeed, we find the admission of a non-zero δ to be rather unwarranted, given that the χ^2 -distribution is very flat for $0 < \delta < 1$ (see Fig. 4 of Ref. [6]).

⁴For example, the Yukawa couplings are uniquely given in terms of the masses.

3 Unitarity Bounds

3.1 Unitarity and $g_{W,Z}$

In a phenomenological study of the Higgs boson, while all its couplings could be varied independently[6], it makes sense to concentrate on the dominant ones. Within the SM, these are the ones with the top quark and the weak gauge bosons. Maintaining the Lorentz structures to be identical to those within the SM, these can be parametrized as in Eq. (1.1), but categorically with $\delta = 0$.

Clearly the effective Lagrangian in Eq. (1.1) would have non-trivial effects on a host of scattering processes, notably on $V_1 V_2 \rightarrow V_3 V_4$ where $V_i = W^\pm, Z$. As is well-known, partial wave unitarity for such scattering processes depends crucially on the couplings being those mandated by $SU(2)_L \otimes U(1)_Y$ invariance alongwith renormalizability. Thus, $g_{W,Z} \neq 1$ could, in principle destroy the same for, say, $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$. This particular scattering proceeds through a set of seven Feynman diagrams, namely a four-point contact interaction, two s -channel diagrams mediated by the γ and the Z (or, in the unbroken symmetry phase, by the W_3), two analogous t -channel ones and, finally, one each of s - and t -channel Higgs-mediated diagrams. With the trilinear (quartic) gauge boson vertices scaling as k^1 (k^0) where k is a typical momentum transfer, and the polarization vector for the longitudinal vector boson going (for large k) as $\epsilon_\mu \sim k_\mu/m_W$, it is obvious that each of the individual pure-gauge diagram contributions to the amplitude goes as $\mathcal{M}_i \sim s^2/m_W^4$. The gauge theory antecedents of the vector-boson self-couplings ensure that the leading terms cancel identically leaving behind a s/m_W^2 behaviour. Once the Higgs-mediated diagrams are included, even the $\mathcal{O}(s)$ contributions cancel, and on integrating the remaining terms over the phase space, one obtains a cross section in consonance with the Froissart bound⁵. Clearly, this cancellation is contingent upon the Higgs couplings being just so, and allowing for $g_W \neq 1$ would result in additional $\mathcal{O}(s \delta g_W/m_W^2)$ contributions from the Higgs-mediated diagram to the amplitude resulting in a bad high-energy behaviour. In Fig. 1, we show the consequent behaviour of the cross sections for a few representative values of g_W . It is easy to ascertain that such a theory loses unitarity at a few TeVs at best⁶ and a new theory needs to be around, and, by implication, *within the reach of the LHC*.

It might be argued, though, that such a deviation in g_W could well be accompanied by others in the gauge boson self couplings, evoking memories of a non-linearly realized symmetry, or at the very least, higher-dimensional terms in an electroweak chiral Lagrangian. While it seems plausible that such correlated deviations could preserve unitarity, it can be seen that simultaneous restoration in all possible channels is difficult to achieve within the ambit of phenomenologically acceptable deviations [14]. However, even if this were to be possible, constraints appear from another sector that we now turn to. This is of particular

⁵Although the presence of a massless photon in the t -channel results in a collinear singularity, this does not violate the Froissart bound. Indeed, this singularity disappears (as it should) when higher order corrections are taken into account.

⁶It is instructive to note that the loss of unitarity occurs not only for $g_W > 1$ (as the fits warrant), but also for $g_W < 1$.

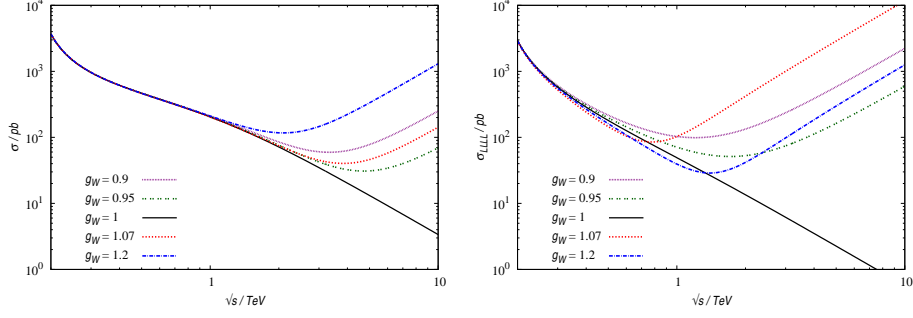


Figure 1. The cross section for $W^+W^- \rightarrow W^+W^-$ as a function of the CM energy, on imposition of a cut $10^\circ \leq \theta \leq 170^\circ$ on the scattering angle. The left (right) panels refer to unpolarized and $W_L^+W_L^- \rightarrow W_L^+W_L^-$ scattering respectively. The individual curves refer to different values of the $WW h$ coupling g_W as normalized to the SM value (see Eq. 1.1).

importance as the deviations $\delta g_{W,Z}$ in the fits $F2$ and $F3$ are relatively small and could shrink further once more data is taken into account.

3.2 Unitarity and g_t

As already mentioned, of the SM particles, the Higgs couples with an unsuppressed strength only to the weak gauge bosons and the top. We have already discussed the consequences of deviations to the former and, now, concentrate on the latter. In analogy to the discussion in the preceding section, this coupling plays a crucial role in processes such as $W^+W^- \rightarrow t\bar{t}$, to which the following diagrams contribute:

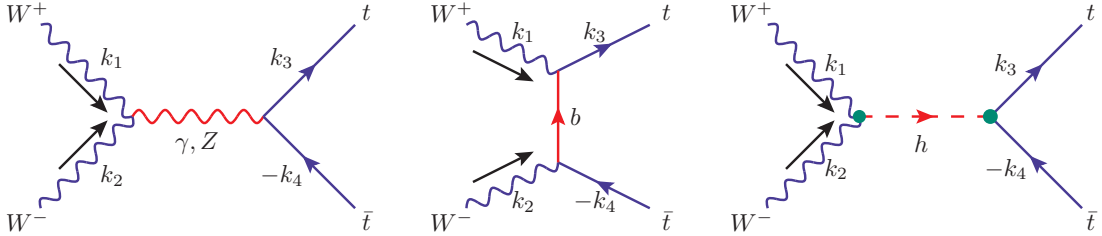


Figure 2. Diagrams contributing to the process $W^+W^- \rightarrow t\bar{t}$.

As can be ascertained from arguments mirroring those in the preceding section, the amplitude that grows most strongly with energy pertains to $W_L^+W_L^-$ annihilation to $t\bar{t}$. Indeed, the Higgs diagram contribution goes as $\mathcal{M}_h \propto g_t g_W m_t \sqrt{s}/m_W^2$ for $\sqrt{s} \gg m_t$. If the coupling g_t deviates from the SM value, the cancellation of the leading term with the non-Higgs diagrams would be imperfect and the amplitude would grow with energy, thereby violating the Froissart bound at some scale. While it may be argued that it is only the combination $g_t g_W$ that comes into play, note that the δg_W needed for the fits can neither compensate for the required δg_t nor is such a large deviation consistent with WW scattering. Similarly, large deviations in the Wtb vertex can be ruled out from the measurements of single-top production at the Tevatron [15] and the LHC [16], as well as from B physics observables such as the mass difference of neutral B meson eigenstates.

This study is best done in terms of the partial wave amplitudes defined as

$$a_\ell \equiv \frac{1}{32\pi} \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) \mathcal{M}(s, \cos\theta; \{m_i, g_i\})$$

where \mathcal{M} is the Lorentz invariant amplitude, θ is the scattering angle and $P_\ell(x)$ the Legendre functions. Unitarity demands that

$$|\text{Re}(a_\ell)| < \frac{1}{2}, \quad \forall \ell$$

and it is the $l = 0$ amplitude a_0 that gives the strongest bound. In particular, the most sensitive probe is given by the amplitude for the particular helicity combination

$$a_0(0, 0, 1, 1) \equiv a_0(W_L^+ W_L^- \rightarrow t_+ \bar{t}_+),$$

with the case for $a_0(0, 0, -1, -1)$ being identical. Denoting the velocities of the particles in the center-of-mass frame by β_W and β_t , one obtains⁷

$$\begin{aligned} a_0(0, 0, 1, 1) = & \frac{-g^2 m_t \sqrt{s}}{128 \pi m_W^2} \left[\frac{\zeta_W}{\beta_W \beta_t} [\beta_W (1 - \beta_W^2) - 2a_W \beta_t] \right. \\ & + \frac{\zeta_W}{2\beta_W \beta_t a_W} [\beta_t (1 + \beta_W^2) + a_W \beta_W (1 - \beta_W^2) - 2a_W^2 \beta_t] \ln \frac{a_W - 1}{a_W + 1} \\ & \left. + 2g_t g_W \beta_t \frac{s - 2m_W^2}{s - m_h^2} \right] \end{aligned} \quad (3.1)$$

where $a_W = (s - 2m_W^2 - 2m_t^2)/(\beta_W \beta_t s)$ and $\zeta_W = |V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2 = 1$. We have assumed here that the gauge couplings of the top quark are unaltered⁸ from those in the SM. While no direct measurement of the $Zt\bar{t}$ vertex is available, once one considers the Wtb vertex to be in consonance with the SM (also indicated to be so by a host of observables such as single top production, top decays as well as B -meson phenomenology), custodial symmetry mandates that the $Zt\bar{t}$ coupling should also be as postulated within the SM.

In a similar vein, we can consider $ZZ \rightarrow t\bar{t}$, to which the following diagrams contribute.

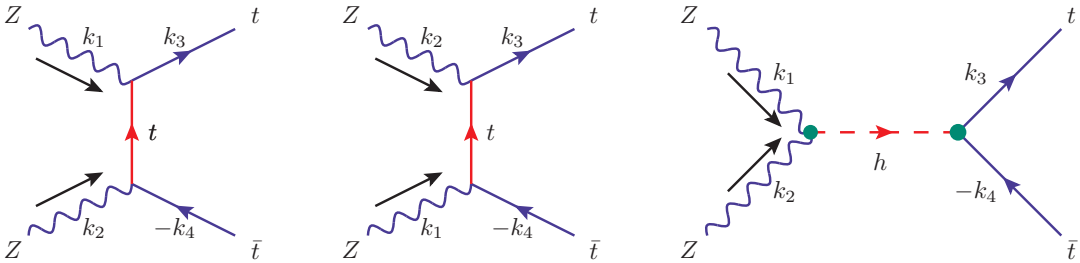


Figure 3. Diagrams contributing to the process $ZZ \rightarrow t\bar{t}$.

⁷The details of the calculations are given in Appendix A.

⁸Note that a significant variation from $\zeta_W = 1$ is strongly disfavoured by constraints from flavour physics.

Once again, $a_0(0, 0, 1, 1)$ proves to be the most sensitive probe. Denoting the coupling of the left–(right–) handed top states with the Z by g_L^{Zt} (g_R^{Zt}), this is given by

$$\begin{aligned}
a_0(0, 0, 1, 1) = & \frac{-m_t \sqrt{s}}{32 \pi m_Z^2} \left[\frac{(g_L^{Zt})^2 + (g_R^{Zt})^2}{\beta_Z \beta_t} [\beta_Z(1 - \beta_Z^2) - 2a_Z \beta_t] \right. \\
& + \frac{(g_L^{Zt})^2 + (g_R^{Zt})^2}{2 \beta_Z \beta_t a_Z} [\beta_t(1 + \beta_Z^2) + a_Z \beta_Z(1 - \beta_Z^2) - 2a_Z^2 \beta_t] \ln \frac{a_Z - 1}{a_Z + 1} \\
& + \frac{4 g_L^{Zt} g_R^{Zt}}{\beta_t} + \frac{g_L^{Zt} g_R^{Zt}}{\beta_Z \beta_t a_Z} [\beta_t(1 + \beta_Z^2) - 2a_Z \beta_Z] \ln \frac{a_Z + 1}{a_Z - 1} \\
& \left. - g_t g_Z \frac{g^2}{2c_W^2} \frac{s - 2m_Z^2}{s - m_h^2} \beta_t \right] \quad (3.2)
\end{aligned}$$

where $a_Z = (s - 2m_Z^2)/(\beta_Z \beta_t s)$.

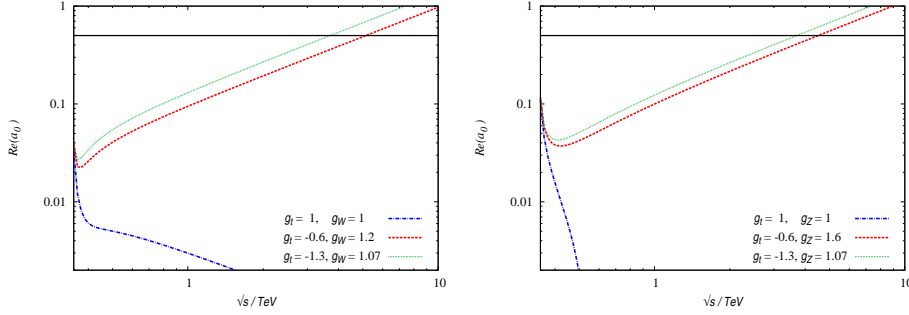


Figure 4. The partial wave amplitude as a function of the center-of-mass energy. The left (right) panel corresponds to $W_L^+ W_L^- \rightarrow t_+ \bar{t}_+$ ($Z_L Z_L \rightarrow t_+ \bar{t}_+$). The curve corresponding to $g_t = g_{W/Z} = 1$ reflect the SM. The black solid line denotes the upper limit from unitarity.

In Fig. 4, we show the variation of the aforementioned a_0 with the center-of-mass energy. As expected, a deviation of the couplings from the SM values cause a significant change in the magnitude of $Re(a_0)$. Indeed, for the most favourable cases of Ref. [6] unitarity would be violated at $\sqrt{s} \gtrsim 4 \text{ TeV}$. In other words, this indicates the maximal energy scale of the effective theory, beyond which a new theory must be operative.

It might be argued, though, that much of the unitarity violation exhibited in Fig. 4 may be caused by the shifts in g_W and g_Z . As discussed in the preceding section, such deviations are strongly disfavoured by considerations involving gauge boson scattering. Indeed, it can be explicitly checked that the violation of unitarity owes itself to a negative value for the product $g_t g_W$ (engendered by a negative g_t). Furthermore, the particular values chosen for the anomalous couplings were dictated by the ‘best fits’ corresponding to a set of data that might soon be overwhelmed by new data. In view of this, it is worthwhile to examine the consequences of having a nonzero δg_t alone, while maintaining all other couplings to their SM values.

In Fig. 5 we display this data in terms of iso- $Re(a_0)$ contours in the $g_t g_V - \sqrt{s}$ plane. Only the white part of the figures bounded by the curves $Re(a_0) = \pm 0.5$ are in consonance with unitarity, and the shaded regions are ruled out. Once again, this shows that even if

all the other couplings were left unmolested, a large deviation in g_t alone would run afoul of unitarity constraints well within a few TeVs. This certainly holds not only for the most favoured values quoted by Ref. [6] but also for a very large fraction of their 95% C.L. allowed regions.

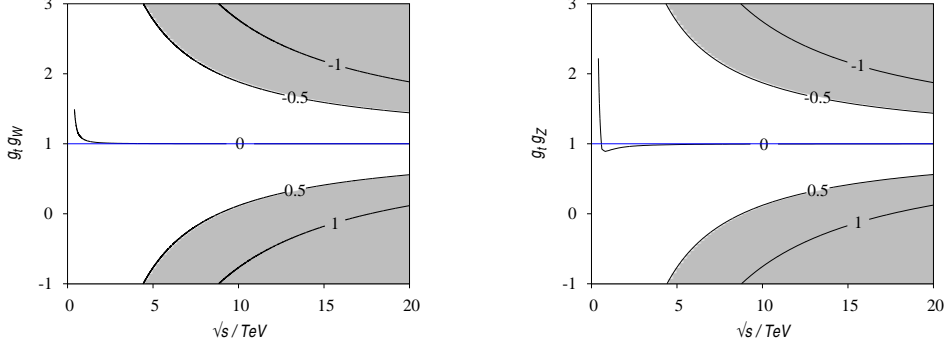


Figure 5. Contours for $Re[a_0(0,0,1,1)]$ in the $g_t g_V - \sqrt{s}$ plane. The left (right) panel corresponds to $W_L^+ W_L^- \rightarrow t_+ \bar{t}_+$ ($Z_L Z_L \rightarrow t_+ \bar{t}_+$). All couplings other than g_t and g_V are held to the SM values.

4 Vacuum Stability

If the consideration of unitarity persuades us to set some cut-off scale to the SM augmented by the anomalous couplings, it is important to check whether the theory respects all other constraints, experimental as well as theoretical. A case in point is the issue of vacuum stability, which demands that the Higgs quartic coupling has to be positive. Once we allow the possibility of a deviation from the SM Yukawa coupling, namely $g_t \neq 1$, the RG evolution for the Higgs quartic coupling λ would be affected too. The RG equation for λ involves only even powers of $g_t h_t$ (where $h_t = \sqrt{2}m_t/v$ is the SM top quark Yukawa coupling), so the sign of g_t is irrelevant, with the evolution depending only on its magnitude.

We use the two-loop β -function for λ , following Refs. [17]. For completeness, they are also quoted in Appendix B. We use the two-loop matching conditions, as given in Ref. [18], to match the data, *viz.*

$$\begin{aligned} m_Z^{\text{pole}} &= 91.1876 \text{ GeV} & \alpha_S(m_Z) &= 0.1184 \\ m_h^{\text{pole}} &= 125.3 \text{ GeV} & \alpha(m_Z) &= 1/127.916 \\ m_t^{\text{pole}} &= 172.9 \text{ GeV} & s_W^2(m_Z) &= 0.23116. \end{aligned} \quad (4.1)$$

to their corresponding values at m_t .

We show the evolution of the scalar quartic coupling λ , and the top Yukawa coupling, in Fig. 6. This shows that the electroweak vacuum might get unstable if $|g_t|$ is even slightly greater than unity, and the point where the instability sets in depends rather sensitively on $|g_t|$. For example, the vacuum becomes unstable at an energy as low as about 10^4 GeV for $g_t = 1.15$. At the one-loop level, the negative term proportional to g_t^4 , coming from a top-mediated box diagram, is responsible for this. Thus, both $F2$ and $F3$ would indicate the presence of new physics $\sim 10^4$ GeV on this account (the unitarity bounds are stronger,

though), while $F1$ seems to be safe. While these shifts parallel those engendered by the errors on the top quark mass measurement itself, there are subtle differences. For one, the shift in $|g_t|$ that the fittings favour are much larger than the experimental errors in m_t (0.6%–1.5% according to various estimates). Moreover, the deployment of the matching conditions in the two cases would differ.

At the same time, we must be cautious about taking these numbers too literally. The calculations hold only if the new physics responsible for the change in the top Yukawa coupling is either above the scale where instability sets in (so that those new degrees of freedom are still frozen), or the effective interaction involves only SM fields but with a new operator structure. In particular, the apparent consistency of $F1$ cannot be depended on, once the physics responsible for unitarity violation is turned on.

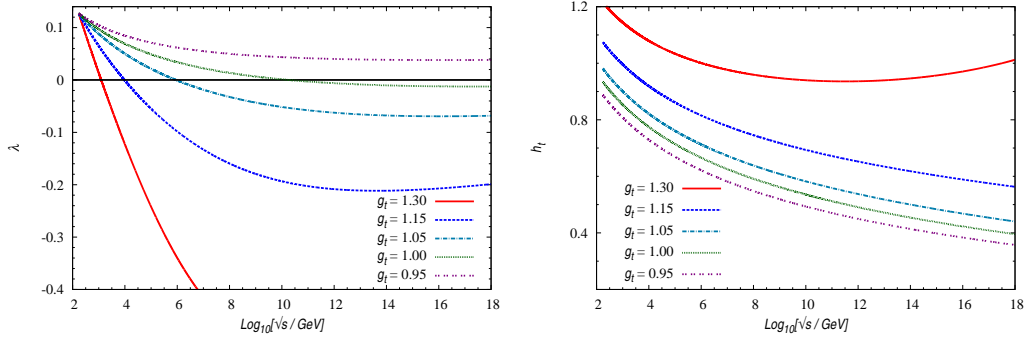


Figure 6. Variation of λ and the top quark Yukawa coupling $g_t h_t$ with CM energy \sqrt{s} , where $h_t = \sqrt{2}m_t/v$.

5 Contemplating Possible Avenues

While we have delineated the problems that beset an effective theory wherein the coupling of the recently glimpsed Higgs-like resonance to the top quark, the W and the Z are reset from the SM values to those obtained from phenomenological best fits, it behoves us to consider possible cures for the same. While an exhaustive treatment is not possible owing to the paucity of independent data, as well as the enormity of the task, we examine some simple alternatives.

5.1 Gauge boson scattering

To begin with, let us consider the effect of $g_{W/Z} \neq 1$. While unequal values for g_W and g_Z do violate custodial symmetry, the most visible consequences appear in electroweak precision observables and can be neutralized by arranging for compensating custodial breaking in other sectors of the theory. Indeed, this has been included in the fitting of Ref. [6]. As for the unitarity violation in gauge boson scattering, curing it would require the introduction of additional contributions to the amplitude. The simplest possibility⁹ would be to postulate

⁹As explained earlier, we do not consider modification of the gauge boson self couplings.

the existence of another scalar, say \tilde{h} , whose couplings parallel those of h in Eq. (1.1), but with the corresponding couplings being \tilde{g}_i , viz.

$$g_W \rightarrow \tilde{g}_W, \quad g_Z \rightarrow \tilde{g}_Z. \quad (5.1)$$

Assuming that this new scalar has a mass $\tilde{M} \gg m_h$, the restoration of unitarity for $\sqrt{s} \gg \tilde{M}$ would require that

$$\tilde{g}_W^2 + g_W^2 = 1, \quad \tilde{g}_Z^2 + g_Z^2 = 1, \quad \tilde{g}_W \tilde{g}_Z + g_W g_Z = 1, \quad (5.2)$$

with the three constraints emanating from considerations of $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, $Z_L Z_L \rightarrow Z_L Z_L$ and $W_L^+ W_L^- \rightarrow Z_L Z_L$ (and crossed processes) respectively. While the requirements might seem trivial at first sight, note that these are actually three conditions on two variables. Moreover, the “best fit points” and, indeed, most of the good fit part of the parameter space found in [6] requires $|g_{W/Z}|^2 > 1$, thereby necessitating negative $|\tilde{g}_{W/Z}|^2$. While this roadblock could be circumvented by postulating a wrong sign for the scalar kinetic term, such a solution brings along its own problems. Note, though, that this would still not guarantee the existence of a simultaneous solution to all three of the above constraints. However, the extent of unitarity violation could be minimized so as to push the scale of violation significantly higher.

The situation simplifies considerably if the scalar \tilde{h} is not an *ad hoc* degree of freedom, but part of another Higgs multiplet that contributes to electroweak symmetry breaking. While only certain representations would guarantee $m_W^2 = m_Z^2 \cos^2 \theta_W$ at the tree-level, it is possible, in principle, to arrange multiple vacuum expectation values for a multitude of representations and carefully tune them to maintain this relation [19, 20]. Obtaining effective $g_{W/Z} > 1$ for at least one such scalar (to be identified with the observed resonance) requires that at least one of these representations must be higher than a doublet [20]. Typically, though, $g_W = g_Z$ would not be maintained. It should be realized that, now, it is not just one new scalar that we would have, but an entire multiplet. This, of course, would change eqs.(5.2) to include additional terms, thereby making it easier to satisfy all three conditions. (This is despite the fact that gauge symmetry would relate several of the new couplings.) The behaviour of the potentially offending cross sections (equivalently, the partial wave amplitudes) would change too; interim phases of growth with \sqrt{s} would be seen, especially as a new Higgs threshold is approached. For very large \sqrt{s} though, the Froissart bound would be seen to be validated.

Yet another way to obtain $g_{W/Z} > 1$ is to postulate a new scalar with non-standard kinetic terms for at least one of the two (the new scalar and h) such that significant kinetic mixing occurs. An example of this is afforded by the radion in warped models [20].

From scalars, we turn our attention to vector bosons as restorers. Unitarizing gauge boson scattering in Higgs-less models through the introduction of new vector bosons has been investigated in Refs. [21, 22]. Clearly, the couplings must satisfy certain conditions. In the presence of a Higgs (albeit with altered couplings), the relations of Ref. [21, 22] have to be altered suitably. The required changes are straightforward, at least as far as the scattering of the SM gauge bosons is concerned. It must be noted, though, that the

introduction of such vector bosons introduces the possibility of a pair of them emanating from, say, $W_L^+ W_L^-$ annihilation. The latter scattering would be associated with its own unitarity violation problems, and, just as in the case of the Higgs-less models, one would have to introduce a tower of such gauge bosons. The tower, in principle, is a infinite one and can be truncated only at the cost of admitting unitarity violation at some scale (or, equivalently, appealing to some ultraviolet completion). Similarly, all the trilinear (and quartic) couplings between this set of vector bosons must satisfy sum rules, the character of which will depend on whether they couple to \tilde{h} .

Finally, it should be noted that this role of unitarity restoration is not restricted to only scalars and vector bosons, but can also be assumed by higher-spin bosons. The inclusion of the latter, though, brings a whole new set of problems to the table, and we desist from any discussion of the same.

5.2 Gauge boson annihilation to fermion pairs

The introduction of a new scalar \tilde{h} could, in principle, restore unitarity for such processes. Once again, denoting the coupling of \tilde{h} to a $t\bar{t}$ pair through a form analogous to Eq. (1.1), but with $g_t \rightarrow \tilde{g}_t$, we can express the conditions for unitarity restoration as

$$\tilde{g}_t \tilde{g}_W + g_t g_W = 1, \quad \tilde{g}_t \tilde{g}_Z + g_t g_Z = 1. \quad (5.3)$$

As before, we are faced with the problem of simultaneous solution of both these constraints, especially once $\tilde{g}_{W/Z}$ are determined from considerations of gauge boson scattering (see preceding section). Of course, with the amplitude here growing only as \sqrt{s}/m_W , a lack of cancellation can be accommodated to a relatively larger degree, yet postponing unitarity violation to $\sqrt{s} > 10$ TeV, or even later.

The main problem, though, is that the best fit requires $g_t g_{W/Z} \sim -1$. This, of course, entails having $\tilde{g}_t \tilde{g}_{W/Z} \sim 2$, or, in other words, rather large couplings for the \tilde{h} . Of particular importance is the fact that the inclusion of scalars in larger representations of $SU(2)$ (and ascribing vacuum expectation values to them) as in the preceding section, not only does not help, but actually worsens the situation. The reason is easy to see. As such large representations would not couple to the top quark (barring non-renormalizable terms), if the wavefunction of the observed resonance were to carry a significant fraction of such a state, its coupling to the top would actually be reduced from the SM value. Thus, one is left with only the possibility of having a large \tilde{g}_t . While this is admissible and can be arranged (in a phenomenological Lagrangian), note that such a \tilde{g}_t would grow rapidly with energy and one would be faced with a Landau pole.

An alternative could be to consider new fermions or gauge bosons which may not couple to our familiar Higgs doublet. However, it is easy to see that this does not help as long as the latter behave canonically.

To summarise, the introduction of a new (set of) scalars with carefully constructed couplings seems to offer the simplest solution to the conundrum. A strictly phenomenological approach, on the other hand, would be given by ascribing form factor behaviour to the

deviations. For example, consider the replacement

$$\delta g_i \rightarrow \delta g_i^0 \left(\frac{2m_h^2}{s+m_h^2} \right)^{n_i}, \quad n_i \geq 1, \quad i = t, W, Z$$

This, clearly would restore unitarity at large energies. This has the further advantage that this permits an examination of the behaviour of the coupling at different energies, thereby permitting some insight into the structure of the deviation once more data is available. Of course, a more generic form factor can be used instead, even correcting for the lack of gauge invariance that the simple-minded expression above entails. This, though, takes us to the regime of electroweak chiral Lagrangians and we shall not delve into it any further.

6 Conclusions

Assuming that the Higgs couplings to the SM fields are arbitrary but consistent with general principles like Lorentz invariance and hermiticity, we tried to see whether the present data gives any hint of new physics beyond the SM. A particularly sensitive probe is offered by considerations of unitarity in gauge boson scattering. We have considered several such scattering amplitudes, for polarized as well as unpolarized gauge bosons, and partial wave unitarity is seen to break down at about $\sqrt{s} \gtrsim 4$ TeV for coupling values preferred by the fits.

Even if this can be prevented by restoring the hWW and hZZ vertices to their SM values (especially since the best fits, anyway, call for only small deviations), we are still faced two rather interesting issues. Indeed, the most important parameter in the study is the top quark Yukawa coupling, whose preferred value has a sign opposite to that of the SM prediction because of the apparent excess of Higgs to diphoton decay rate. We explored the consequences of such a wrong-sign coupling.

There are two places where the wrong-sign Yukawa coupling can play havoc. The first is the unitarity in gauge boson annihilation to a $t\bar{t}$ pair. This effect can be traced to a term in the scattering amplitude which is proportional to the product of the top quark Yukawa coupling and the hVV coupling. With the sign flip of this term, the amplitudes grow up instead of going down and one sees unitarity violation at $\sqrt{s} \gtrsim 5$ TeV. Thus, this indicates some new physics which takes over at a few TeV scale and restores unitarity as well as gauge invariance, which is apparently broken by Eq. (2.3).

The second place is the stability of the electroweak vacuum. The Higgs quartic coupling λ becomes negative if the magnitude of the top Yukawa coupling increases even a little from its SM value (only $|g_t|$ is important here, and not the sign of g_t). The point where the vacuum becomes unstable is a sensitive function of g_t , but for our benchmark points, occur between 1 and 10 TeV, a region already indicated by the unitarity violation. Again, this asks for some new degrees of freedom, which couple to the Higgs and make the vacuum stable (so these should better be bosonic in nature). Of course, whether the cutoff of the theory is at the Planck scale or at a few TeVs does not affect the $h \rightarrow \gamma\gamma$ rate as this must always be finite.

It might be argued that the departures from standard couplings as suggested by the data are based on *global analyses*, where other couplings are simultaneously assuming non-standard values. This could be construed to mean that a complete analysis will have to take into account the role of the other modified couplings in the evolution of λ as well as in ensuring unitarity in scattering phenomena. While, as a principle, this is certainly true, note that our analysis has included all of the relevant dimension-4 terms that can be written down in terms of the SM fields alone. Although the inclusion of subdominant terms would alter the quantitative details of our conclusions, no qualitative change would be brought about.

Thus, if the trend—in particular the excess in diphoton channel—persists in the new data, this might lead to some indirect evidence of new physics which is lurking close.

Note added. While the work was being completed, we became aware of a similar work in progress [23]. Similarly, the very recent announcements by the ATLAS and CMS collaborations [24] do not change the fit materially, thereby validating our conclusions.

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A Momenta, Polarizations and Helicity Amplitudes

In our calculations we have denoted the momenta of the particles as follows

$$\begin{aligned} k_1 &= \frac{\sqrt{s}}{2}(1, 0, 0, \beta_V); \quad k_2 = \frac{\sqrt{s}}{2}(1, 0, 0, -\beta_V); \\ k_3 &= \frac{\sqrt{s}}{2}(1, \beta_t s_\theta, 0, \beta_t c_\theta); \quad k_4 = \frac{\sqrt{s}}{2}(1, -\beta_t s_\theta, 0, -\beta_t c_\theta), \end{aligned} \quad (\text{A.1})$$

where \sqrt{s} is the CM energy, $\beta_V = \sqrt{1 - 4m_V^2/s}$ and $\beta_t = \sqrt{1 - 4m_t^2/s}$. $V = W^\pm, Z$ in the appropriate cases.

The polarization vectors have been denoted as

$$\epsilon_{k_1}^{\hat{\lambda}} = \frac{1}{\sqrt{2}}(-\hat{\lambda}\epsilon_1 - i\hat{\lambda}^2\epsilon_2) + (1 - \hat{\lambda}^2)\epsilon_3; \quad \epsilon_{k_2}^{\hat{\lambda}} = \frac{1}{\sqrt{2}}(\hat{\lambda}\epsilon_1 - i\hat{\lambda}^2\epsilon_2) + (1 - \hat{\lambda}^2)\epsilon_4; \quad (\text{A.2})$$

where $\hat{\lambda} = 0$ corresponds to the longitudinal and $\hat{\lambda} = \pm$ are the transverse polarizations. ϵ_i are as follows

$$\epsilon_1 = (0, 1, 0, 0); \quad \epsilon_2 = (0, 0, 1, 0); \quad \epsilon_3 = \frac{\sqrt{s}}{2m_V}(\beta_V, 0, 0, 1); \quad \epsilon_4 = \frac{\sqrt{s}}{2m_V}(\beta_V, 0, 0, -1). \quad (\text{A.3})$$

The helicity states of top quarks are given by

$$\chi_+(k_3) = \begin{pmatrix} c_{\theta/2} \\ s_{\theta/2} \end{pmatrix}, \quad \chi_-(k_3) = \begin{pmatrix} -s_{\theta/2} \\ c_{\theta/2} \end{pmatrix}; \quad \chi_+(k_4) = \begin{pmatrix} s_{\theta/2} \\ -c_{\theta/2} \end{pmatrix}, \quad \chi_-(k_4) = \begin{pmatrix} c_{\theta/2} \\ s_{\theta/2} \end{pmatrix} \quad (\text{A.4})$$

From (A.4) we can get the 4-component Dirac spinors as

$$u(p, \hat{\lambda}) = \begin{pmatrix} \omega_{-\hat{\lambda}}(p) \chi_{\hat{\lambda}}(\hat{\mathbf{p}}) \\ \omega_{\hat{\lambda}}(p) \chi_{\hat{\lambda}}(\hat{\mathbf{p}}) \end{pmatrix}; \quad v(p, \hat{\lambda}) = \begin{pmatrix} -\hat{\lambda} \omega_{\hat{\lambda}}(p) \chi_{-\hat{\lambda}}(\hat{\mathbf{p}}) \\ \hat{\lambda} \omega_{-\hat{\lambda}}(p) \chi_{-\hat{\lambda}}(\hat{\mathbf{p}}) \end{pmatrix}. \quad (\text{A.5})$$

Here we have defined $\omega_{\hat{\lambda}}(p) = \sqrt{E + \hat{\lambda}|\mathbf{p}|}$.

Using the momenta from (A.1), polarizations and helicity states from (A.2), (A.4) respectively and the taking the effective Lagrangian of Eq.(1.1), we get the helicity amplitudes for $W^+W^- \rightarrow t\bar{t}$ as

$$\mathcal{M}_{0011}^{\gamma s} = -\frac{2}{3}g^2 s_W^2 \frac{1}{s} \frac{m_t}{m_W^2} \beta_W \sqrt{s} (s + 2m_W^2) c_\theta \quad (\text{A.6})$$

$$\mathcal{M}_{0011}^{Zs} = -g^2 \left[1 - \frac{8}{3}s_W^2 \right] \frac{1}{s - m_W^2} \frac{m_t}{4m_W^2} \beta_W \sqrt{s} (s + 2m_W^2) c_\theta \quad (\text{A.7})$$

$$\mathcal{M}_{0011}^t = -\frac{g^2}{2} |V_{tb}|^2 \frac{1}{t} \frac{m_t}{8m_W^2} s^{3/2} [\beta_W (1 - \beta_W^2) c_\theta - 2\beta_t c_\theta^2 + \beta_t (1 + \beta_W^2)] \quad (\text{A.8})$$

$$\mathcal{M}_{0011}^h = -g_t g_W \frac{g^2 m_t}{2} \frac{1}{s - m_h^2} \frac{1}{2} \beta_t \sqrt{s} \left(\frac{s}{m_W^2} - 2 \right) \quad (\text{A.9})$$

Similarly we get the helicity amplitudes for $ZZ \rightarrow t\bar{t}$ as

$$\begin{aligned} \mathcal{M}_{0011}^t &= -[(g_L^{Zt})^2 + (g_R^{Zt})^2] \frac{1}{t - m_t^2} \frac{m_t}{8m_Z^2} s^{3/2} [\beta_Z (1 - \beta_Z^2) c_\theta - 2\beta_t c_\theta^2 + \beta_t (1 + \beta_Z^2)] \\ &+ g_L^{Zt} g_R^{Zt} \frac{1}{t - m_t^2} \frac{m_t}{4m_Z^2} s^{3/2} [\beta_t (1 + \beta_Z^2) - 2\beta_Z c_\theta] \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \mathcal{M}_{0011}^u &= -[(g_L^{Zt})^2 + (g_R^{Zt})^2] \frac{1}{u - m_t^2} \frac{m_t}{8m_Z^2} s^{3/2} [-\beta_Z (1 - \beta_Z^2) c_\theta - 2\beta_t c_\theta^2 + \beta_t (1 + \beta_Z^2)] \\ &+ g_L^{Zt} g_R^{Zt} \frac{1}{u - m_t^2} \frac{m_t}{4m_Z^2} s^{3/2} [\beta_t (1 + \beta_Z^2) + 2\beta_Z c_\theta] \end{aligned} \quad (\text{A.11})$$

where $g_L^{Zt} = -\frac{g}{2c_W} \left(1 - \frac{4}{3}s_W^2 \right)$, $g_R^{Zt} = \frac{g}{2c_W} \frac{4}{3}s_W^2$. $s_W^2 = \sin^2 \theta_W$, $c_W^2 = \cos^2 \theta_W$ and θ_W , the Weinberg angle.

$$\mathcal{M}_{0011}^h = -g_t g_Z \frac{g^2 m_t}{4c_W^2} \frac{1}{s - m_h^2} \beta_t \sqrt{s} \left(\frac{s}{m_Z^2} - 2 \right) \quad (\text{A.12})$$

B Beta Functions

We give the beta functions used in our calculation from the appendix of Ref. [25]. The beta function for a generic coupling X is given as:

$$\mu \frac{dX}{d\mu} = \beta_X = \sum_i \frac{\beta_X^{(i)}}{(16\pi^2)^i}. \quad (\text{B.1})$$

The beta functions are given, above m_t but below any new degrees of freedom, by [17]:

$$\begin{aligned} \beta_\lambda^{(1)} &= 24\lambda^2 + 12\lambda h_t^2 - 6h_t^4 - 3\lambda g_1^2 - 9\lambda g_2^2 + \frac{3}{4}g_2^4 + \frac{3}{8}(g_1^2 + g_2^2)^2, \\ \beta_{h_t}^{(1)} &= \frac{9}{2}h_t^3 - \frac{17}{12}h_t g_1^2 - \frac{9}{4}h_t g_2^2 - 8h_t g_3^2, \\ \beta_{g_1}^{(1)} &= \frac{41}{6}g_1^3, \quad \beta_{g_2}^{(1)} = -\frac{19}{6}g_2^3, \quad \beta_{g_3}^{(1)} = -7g_3^3, \\ \beta_\lambda^{(2)} &= -312\lambda^3 - 144\lambda^2 h_t^2 - 3\lambda h_t^4 + 36\lambda^2 g_1^2 + 108\lambda^2 g_2^2 + 80\lambda h_t^2 g_3^2 + \frac{45}{2}\lambda h_t^2 g_2^2 + \frac{85}{6}\lambda h_t^2 g_1^2 \\ &\quad - \frac{73}{8}\lambda g_2^4 + \frac{39}{4}\lambda g_2^2 g_1^2 + \frac{629}{24}\lambda g_1^4 + 30h_t^6 - 32h_t^4 g_3^2 - \frac{8}{3}h_t^4 g_1^2 - \frac{9}{4}h_t^2 g_2^4 \\ &\quad + \frac{21}{2}h_t^2 g_2^2 g_1^2 - \frac{19}{4}h_t^2 g_1^4 + \frac{305}{16}g_2^6 - \frac{289}{48}g_2^4 g_1^2 - \frac{559}{48}g_2^2 g_1^4 - \frac{379}{48}g_1^6, \\ \beta_{h_t}^{(2)} &= 6\lambda^2 h_t - 12\lambda h_t^3 - 12h_t^5 + \frac{131}{16}h_t^3 g_1^2 + \frac{1187}{216}h_t g_1^4 - \frac{3}{4}h_t g_1^2 g_2^2 + \frac{19}{9}h_t g_1^2 g_3^2 + \frac{225}{16}h_t^3 g_2^2 \\ &\quad - \frac{23}{4}h_t g_2^4 + 9h_t g_2^2 g_3^2 + 36h_t^3 g_3^2 - 108h_t g_3^4, \\ \beta_{g_1}^{(2)} &= -\frac{17}{6}g_1^3 h_t^2 + \frac{199}{18}g_1^5 + \frac{9}{2}g_1^3 g_2^2 + \frac{44}{3}g_1^3 g_3^2, \\ \beta_{g_2}^{(2)} &= -\frac{3}{2}h_t^2 g_2^3 + \frac{3}{2}g_1^2 g_2^3 + \frac{35}{6}g_2^5 + 12g_2^3 g_3^2, \\ \beta_{g_3}^{(2)} &= -2h_t^2 g_3^3 + \frac{11}{6}g_1^2 g_3^3 + \frac{9}{2}g_2^2 g_3^3 - 26g_3^5. \end{aligned} \quad (\text{B.2})$$

In the above λ is the quartic Higgs coupling, h_t , the top Yukawa coupling, g_1, g_2 and g_3 are the $U(1)_Y, SU(L)_L$ and $SU(3)_C$ couplings respectively.

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